B.Sc. Part—I (Semester—I) (CBCS) (New) Examination

MATHEMATICS

(Differential & Integral Calculus)

Paper—II (DSC–II)

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory, attempt it once only.

1. Choose correct alternatives :

(1) $|x - x_0| < \delta$ represents :

(a) $x_0 - \delta < x < x_0 + \delta$ (b) $x_0 + \delta < x < x_0 - \delta$

(c) $x_0 - \delta \le x < x_0 + \delta$ (d) $x_0 - \delta < x \le x_0 + \delta$

(2) The value of $\lim_{x \to 0} \frac{\sin x}{x}$ is :

(a) 0 (b) 1

(c) ∞ (d) None of these

(3) If f(x) is differentiable at $x = x_0$ then :

(a) f(x) is not continuous at $x = x_0$

(b) f(x) has removable discontinuity at $x = x_0$

(c) f(x) has simple discontinuity at $x = x_0$

(d) f(x) is continuous at $x = x_0$

(4) If f is continuous on a closed interval, then it is _____ on that interval.

(a) Unbounded (b) Bounded

(c) Closed bounded (d) Open bounded

(5) If $y = e^{-3x}$ then y_{11} = _____.

(a) $-3^{11}e^{-3x}$ (b) $3^{11}e^{-3x}$

(c) $-e^{-3x}$ (d) None of these

(6) The value of $\lim_{x \to 0} \frac{2x^3 - 3x^2 + 1}{3x^5 - 5x^3 + 2}$ = _____.

(a) $\frac{1}{2}$ (b) $\frac{1}{5}$

(c) $\frac{1}{3}$ (d) None of these

(7) The series : $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$ is the expansion of function :

(a) sin x

(b) sinh x

(c) cos x

(d) cosh x

(8) The series $f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + ... + \frac{x^{n-1}}{(n-1)!} f^{(n-1)}(0) + ...$ is called :

(a) Taylor's series

(b) Maclaurin's series

(c) Lagrange's series

(d) None of these

(9) If $In = \int \cos^n x \, dx$ then the reduction formula for In is :

(a) $In = \frac{1}{n} \cos^{n-1} x \cdot \sin x + \frac{n-1}{n} In - 2$

(b) $In = -\frac{1}{n} \sin^{n-1} x \cdot \cos x + \frac{n-1}{n} In - 2$

(c) $In = -\frac{1}{n} \cos^{n-1} x \cdot \sin x + \frac{n-1}{n} In - 2$

(d) $In = \frac{1}{n} \cos^{n-1} x \cdot \sin x + \frac{n-1}{n} In - 2$

(10) The value of $\int_0^{\pi/2} \cos^7 x \, dx$ is :

(a) $\frac{16}{35}$

(b) $\frac{16}{21}$

(c) $\frac{35}{16}$

(d) $\frac{21}{16}$

1×10=10

UNIT—I

2\. (a) If $\lim_{x \to x_0} f(x) = \ell$ and $\lim_{x \to x_0} g(x) = m$ and these limits exists. Then prove that :

$$\lim_{x \to x_0} [f(x) \cdot g(x)] = \ell \cdot m$$

6

OR

(b) (i) Prove that, the limit of a function at a point if it exists, is unique.

(ii) Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$.

4+2

(c) Show that $\lim_{x \to 3} x^2 = 9$, using $\varepsilon - \delta$ definition of limit. 4

OR

(d) Show that $\lim_{x \to 0} \frac{\sin x}{x} = 1$. 4

UNIT—II

3. (a) Show that if

$$f(x) = \begin{cases} \frac{e^{1/x}}{1 + e^{1/x}} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$$

then f(x) has a simple discontinuity at x = 0. 6

OR

(b) Define continuous function and prove that the function f(x) defined by $f(x) = \frac{1}{x}$, $x \neq 0$ is continuous for all real values of x. 1+5

(c) Prove that if $f(x) = \sqrt{x - 2}$ for $2 \leq x \leq 4$, then f(x) is continuous in the interval. 4

OR

(d) Prove that the function f(x) = sin x is uniformly continuous on $(-\infty, \infty)$. 4

UNIT—III

4. (a) Prove that if a function f(x) is derivable at point x = a then it is continuous at x = a. But converse is not true. 6

OR

(b) (i) Evaluate : $\lim_{x \to \frac{\pi}{2}} \frac{\log\left(x - \frac{\pi}{2}\right)}{\tan x}$

(ii) Evaluate : $\lim_{x \to 0} x \cdot \log x$. 3+3

(c) If y = log (ax + b), then prove that :

$$y_n = \frac{(-1)^n (n-1)!\, a^n}{(ax + b)^n}$$ 4

OR

(d) If $y = x^n \cdot \log x$ then prove that :

$$y_{n+1} = \frac{n!}{x}$$ 4

UNIT—IV

5. (a) State and prove Lagrange's Mean value theorem. 6

OR

(b) State and prove Rolle's theorem. 6

(c) Find the Taylor's series expansion for the function $f(x) = x^4 + x - 2$ at $a = 1$. 4

OR

(d) Obtain the Maclaurin's series expansion for $f(x) = \log(1 + x)$. 4

UNIT—V

6. (a) Integrate $\int \frac{x^3 + 3}{\sqrt{x^2 + 1}} dx$. 6

OR

(b) If $I_n = \int \sin^n x \, dx$, then prove that :

$$I_n = -\frac{1}{n}\sin^{n-1} x \cdot \cos x + \frac{n-1}{n} I_{n-2}$$ 6

(c) Show that $\int_0^a \frac{x^4 dx}{\sqrt{a^2 - x^2}} = \frac{3a^4\pi}{16}$. 4

OR

(d) Evaluate : $\int_0^{\pi/4} \sin^4 x \, dx$. 4